

## QUADRUPOLE TORSIONAL RIGIDITY MEASUREMENT

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### ABSTRACT

An initial measurement of the torsional rigidity of the Doubler Quadrupole Collared Coil Assembly has been made.

### MEASUREMENT APPARATUS

Figure 1 shows the measuring apparatus. Measurements were made by rigidly clamping one end of a welded, epoxied collared coil assembly (QB-8) to a steel assembly table using an energpac hydraulic cylinder exerting a force on four, 3/4" diameter pieces of drill rod (Fig. 2). The other end of the coil rested on thin, oiled shims and was constrained from moving laterally but was free to rotate (Fig. 3). A strap wrench wrapped around the free end of the coil was used to apply known torque by hanging weights from the wrench.

Measurements of coil rotation were made using a dial indicating height gauge referenced to a 1/2" diameter piece of drill rod which was slid along the notches on each side of the coil. This vertical displacement was used to determine actual coil rotation.

### DATA

Figure 4 shows the coil rotation as a function of distance from the fixed end. The numbers represent net twist, i.e., the inherent twist in the coil has been subtracted out. One fact not shown in the graphs is that the coil retained part of its twist after releasing the

torque; applying a torque in the opposite direction then caused the coil to unwind and return to its original position. This may have been due to the fact that the coil was not supported on a frictionless roller at its free end.

### CONCLUSIONS

The quantity of interest is the torsional rigidity, which allows finding the angle of twist per unit length:

$$\theta = \frac{T}{GJ} = \text{Angle of twist per inch of length (radians)}$$

where

$T$  = torque (in lb)

$G$  = shear modulus of elasticity (psi)

$J$  = polar moment of inertia (in<sup>4</sup>)

In the above expression  $\theta$ ,  $T$  can be measured and  $J$  can be calculated allowing  $G$  to be calculated. For a cylinder:

$$J = \frac{\pi}{2} (r_o^4 - r_i^4) = \frac{\pi}{2} (2.933^4 - 1.750^4) \text{ in}^4 = 101.5 \text{ in}^4$$

From the data taken:

$$\theta_c/T = .014 \text{ m rad/ft lb}$$

$$\theta_{cc}/T = .011 \text{ m rad/ft lb}$$

where

$\theta_c$  = clockwise coil twist (over 54 inch length)

$\theta_{cc}$  = counterclockwise coil twist (over 54 inch length)

The shear modulus of elasticity can now be found

$$G_c = \frac{TL}{\theta_c J} = 4.56E5 \text{ psi}$$

$$G_{cc} = \frac{TL}{\theta_{cc} J} = 5.80E5 \text{ psi}$$

where

$G_c$  = clockwise shear modulus of elasticity

$G_{cc}$  = counterclockwise shear modulus of elasticity

$L$  = length of coil being twisted

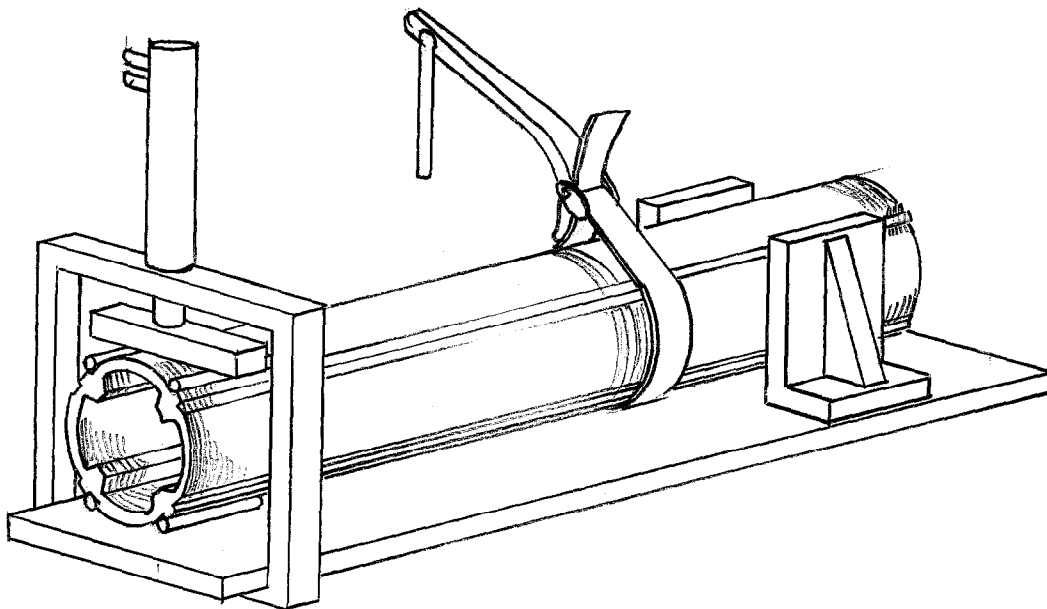


FIGURE 1: COIL TWIST MEASUREMENT APPARATUS

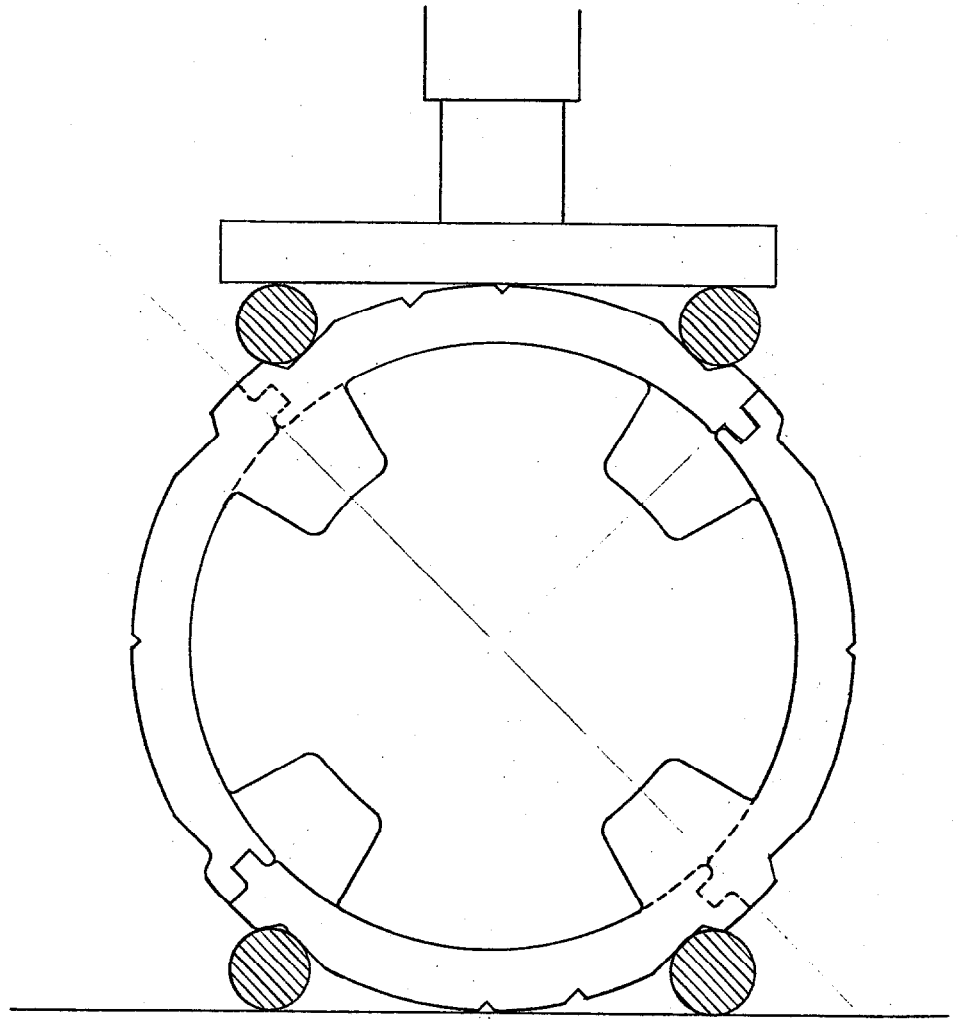


FIGURE 2 : CLAMPED END OF COIL

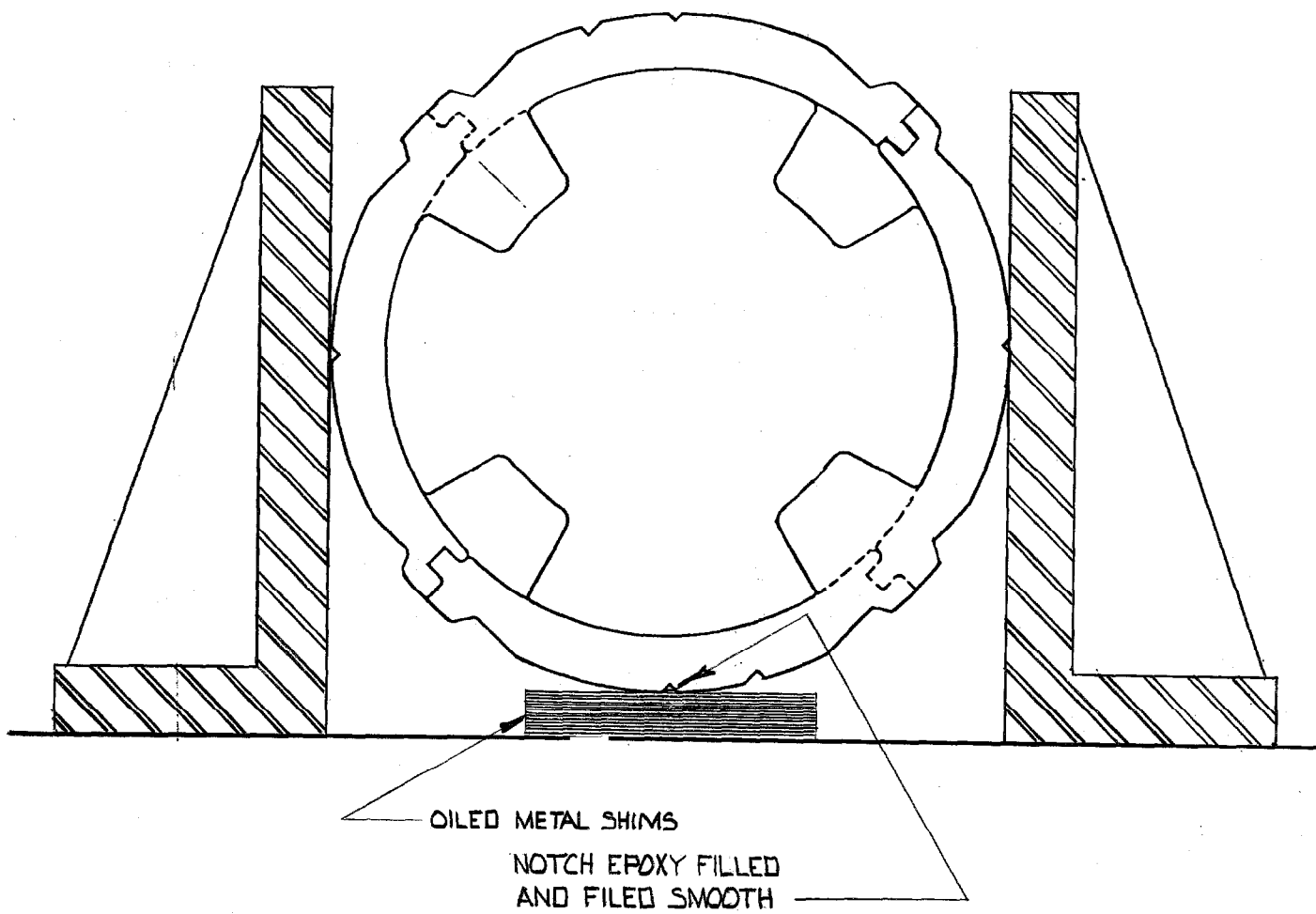


FIGURE 3:  
SUPPORT OF FREE END OF COIL

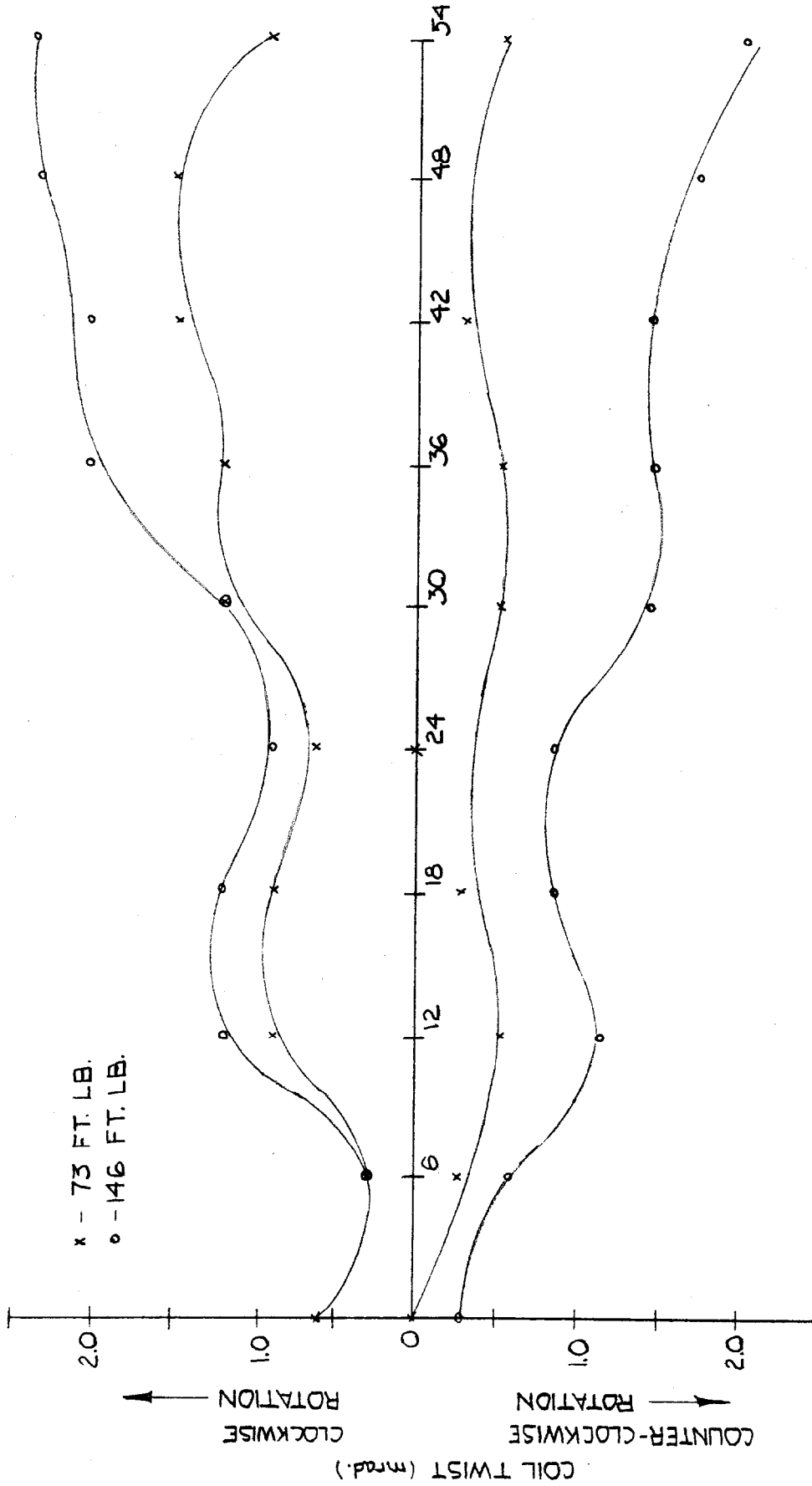


FIG. 4: COIL TWIST VS. DISTANCE FROM FIXED END (INCHES)